# Development of 3-D FBR Heterogeneous Core Calculation Method Based on Characteristics Method 

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#### Abstract

A new 3-D transport calculation method taking into account the heterogeneity of fuel assemblies has been developed by combining the characteristics method and the nodal transport method. In the axial direction the nodal transport method is applied, and the characteristics method is applied to take into account the radial heterogeneity of fuel assemblies. The numerical calculations have been performed to verify 2-D radial calculations of FBR assemblies and partial core calculations. Results are compared with the reference Monte-Carlo calculations. A good agreement has been achieved. It is shown that the present method has an advantage in calculating reaction rates in a small region.


## 1. Introduction

The core calculations of fast reactors are usually performed in terms of multi-group diffusion and transport theory with assembly homogenized cross sections and diffusion coefficient. The diffusion calculations are recently carried out using the nodal diffusion theory. In the theory the assembly homogenized cross sections and the flux
discontinuity factors are utilized. In Japan, however, the flux discontinuity factor has not been utilized in fast reactor analysis. The effective cross sections that preserve the assembly averaged reaction rate in a color set including different types of assemblies have been used in the diffusion calculations without using the flux discontinuity factor.
To consider the transport correction, which cannot be neglected in fast reactor analysis, in 3-D geometry, the 3-D transport codes such as TRITAC ${ }^{(1)(2)}$ and NSHEX $^{(3)(4)}$ were developed in Japan.
TRITAC is the 3-D XYZ Sn based on the finite difference method, and the diffusion synthetic acceleration technique is utilized to shorten computing time. The code has been successfully applied to the fast critical assemblies $Z_{P P R}{ }^{(5)}$ to analyze the critical experiment plan JUPITER, and to the prototype fast reactor MONJU for detailed analysis.
NSHEX is the 3-D Hex-Z nodal transport code with the explicit expression of neutron angular distribution by the Sn quadrature.

These codes can estimate the whole core transport correction. However, the local transport correction is difficult to estimate because each assembly is homogenized, and there is no technique to utilize the flux discontinuity factor within the framework of transport theory.

Furthermore, in MONJU or JOYO, there are special assemblies loaded at some in the core to measure the reaction rate of several elements. These assemblies are composed of fuel pins and special capsules which contain dosimeter foils.
To analyze the reaction rates in such a small region it is necessary to use multi-dimensional transport codes which can treat the heterogeneity of assemblies.
The purpose of this report is to develop a 3-D transport method treating the heterogeneity of assemblies. In Chap. 2 we will develop a new method by combining the characteristics method ${ }^{(6)(7)}$ and the nodal transport method. Numerical results for 2-D infinite assembly calculations and 2-D color set calculations for the present method are compared with the reference Monte-Carlo method in Chap.3.

## 2. 3-D Transport Theory Method

Let us consider the following multi-group transport equation in 3-D geometry.

$$
\begin{equation*}
\Omega_{x} \frac{d \phi^{g}(\rho, \breve{\Omega})}{d x}+\Omega_{y} \frac{d \phi^{g}(\rho, \breve{\Omega})}{d y}+\Omega_{z} \frac{d \phi^{g}(\rho, \breve{\Omega})}{d z}+\Sigma_{t}^{g}(\rho) \phi(\rho, \varrho)=Q^{g}(\rho, \varrho) \tag{1}
\end{equation*}
$$

where the source term is expressed by

$$
\begin{equation*}
Q^{g}(\rho, \Omega)=\int d \Omega^{\prime} \sum_{g} \Sigma_{s}^{g^{\prime} \rightarrow g}\left(\rho, \Omega^{\prime} \rightarrow \Omega^{\varrho}\right) \phi^{g^{\prime}}\left(\rho, \Omega^{\prime}\right)+\frac{\chi^{g}}{4 \pi} \int d \Omega^{\prime} \sum_{g^{\prime}} v \Sigma_{f}^{g^{\prime}}(\rho) \phi^{g^{\prime}}\left(\rho, \Omega^{\prime}\right)( \tag{2}
\end{equation*}
$$

The hexagonal-Z core geometry is divided into nodes. Each hex-Z node corresponding finite height of an assembly includes heterogeneous pin arrays. By integrating Eq.(1) about $Z$ over $Z_{k}<Z<Z_{k+1}$, one obtains

$$
\begin{equation*}
\Omega_{x} \frac{d \phi^{g}\left(\rho, \Omega_{2}\right)}{d x}+\Omega_{y} \frac{d \phi^{g}(\rho, \breve{\Omega})}{d y}+\Sigma_{k}^{g}(x, y) \phi_{k}^{g}(x, y, \Omega)=Q_{k}^{g}(x, y, \Omega)-L_{k}^{g}(x, y, \Omega) \tag{3}
\end{equation*}
$$

where

$$
\begin{gather*}
\phi_{k}^{g}(x, y, \Omega) \equiv \int_{z_{k}}^{z_{k+1}} d z \phi^{g}(\rho, \Omega)  \tag{4}\\
Q_{k}^{g}(x, y, \Omega)=\int_{z_{k}}^{z_{k+1}} d z Q^{g}(\rho, \Omega)  \tag{5}\\
L_{k}^{g}(x, y, \stackrel{\Omega}{\Omega})=\Omega_{z}\left\{\phi\left(x, y, z_{k+1}, \stackrel{\mu}{\Omega}\right)-\phi\left(x, y, z_{k}, \Omega_{)}\right)\right\} \tag{6}
\end{gather*}
$$

Equation (3) is solved by the characteristics method. Using the distances taken al ong a neutron flight, Eq.(4) can be written as

$$
\begin{equation*}
\frac{d \phi_{k, i}^{g}}{d s_{k}}+\Sigma_{i}^{g} \phi_{k, i}^{g}=Q_{k, i}^{\prime g}(=Q-L) \tag{7}
\end{equation*}
$$

The outgoing neutron from a region is given by

$$
\begin{equation*}
\phi_{k, i}^{g}(\text { out })=\phi_{k, i}^{g}(\text { in }) \exp \left(-\Sigma_{i}^{g} l\right)+\frac{Q_{k, i}^{\prime g}}{\Sigma_{i}^{g}}\left(1-\exp \left(-\Sigma_{i}^{g} l\right)\right) \tag{8}
\end{equation*}
$$

The pass length in a region is called as segment (Fig.1). The segment averaged angular flux is given by

$$
\begin{equation*}
\bar{\phi}_{k, i}^{g}=\frac{\int_{0}^{l} \phi_{k, i}^{g} d s_{k}}{\int_{0}^{l} d s_{k}}=\frac{Q_{k, i}^{\prime g}}{\Sigma_{i}^{g}}+\frac{\phi_{k, i}^{g}(\text { in })-\phi_{k, i}^{g}(\text { out })}{\sum_{i}^{g} l} \tag{9}
\end{equation*}
$$

The region-averaged angular flux is calculated by

$$
\begin{equation*}
\Phi_{k, i}^{g}=\frac{\sum_{m} \bar{\phi}_{k, i, m}^{g} l_{k, i, m} \delta A_{k, m}}{\sum_{m} l_{k, i, m} \delta A_{k, m}} \tag{10}
\end{equation*}
$$

and the averaged flux is calculated by

$$
\begin{equation*}
\bar{\Phi}_{i}^{g}=\sum_{k} \omega_{k} \Phi_{k, i}^{g} \tag{11}
\end{equation*}
$$

where $\omega$ is the weight for direction k .


Fig 1 Neutron pass in Characteristics Method
A computer code BACH(Beneficial and Advanced transport Characteristics method for Hexagonal cell) has been developed based on the above method. In the code the combination of heterogeneous hexagonal assemblies can be handled as shown in Fig.2.


Fig 2 Combination of heterogeneous hexagonal cells

## 3. Numerical Calculations and Discussions

The BACH code has been applied to two problems, infinite assembly calculations and color set (mini-core) calculations. The calculations were carried out in 70 groups using the cross section set from the JENDL-3.2 library ${ }^{(8)}$.

### 3.1 Assembly Calculation

The calculation model is show in Fig.3. The central 169 pins have Pu fuel of enrichment of $20.03 \mathrm{wt} \%$, and surrounded by a wrapper tube of 0.3 cm thickness. The eigenvalue $k_{\infty}$ is calculated by varying the pass line distance and the polar angle division. The division number of azimuthal angle was fixed to 360 .


Fig 3 Calculation model
The $k_{\infty}$ results are shown in Table 1

Table 1 Results of $k_{\infty}$ for various calculations with different pass line distance and polar angle division number

|  | Polar angle division number |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Pass line distance (cm) | 6 | 3 | 2 | 1 |
| 0.01 | 1.34927 | 1.34927 | 1.34924 | 1.34909 |
|  | - | $+0.000 \% \Delta \mathrm{k} / \mathrm{k}$ | $-0.002 \% \Delta \mathrm{k} / \mathrm{k}$ | $-0.013 \% \Delta \mathrm{k} / \mathrm{k}$ |
| 0.05 | 1.34928 | 1.34929 | 1.34926 | 1.34910 |
|  | $+0.001 \% \Delta \mathrm{k} / \mathrm{k}$ | $+0.001 \% \Delta \mathrm{k} / \mathrm{k}$ | $-0.001 \% \Delta \mathrm{k} / \mathrm{k}$ | $-0.013 \% \Delta \mathrm{k} / \mathrm{k}$ |
| 0.1 | 1.34919 | 1.34920 | 1.34917 | 1.34901 |
|  | $-0.006 \% \Delta \mathrm{k} / \mathrm{k}$ | $-0.006 \% \Delta \mathrm{k} / \mathrm{k}$ | $-0.007 \% \Delta \mathrm{k} / \mathrm{k}$ | $-0.019 \% \Delta \mathrm{k} / \mathrm{k}$ |
| 0.5 | 1.34999 | 1.35000 | 1.34998 | 1.34981 |
|  | $+0.053 \% \Delta \mathrm{k} / \mathrm{k}$ | $+0.054 \% \Delta \mathrm{k} / \mathrm{k}$ | $+0.052 \% \Delta \mathrm{k} / \mathrm{k}$ | $+0.040 \% \Delta \mathrm{k} / \mathrm{k}$ |

The reference Monte-Carlo result calculated by GMVP is $1.34926 \pm 0.0113 \%$. The BACH result with pass line distance of 0.01 cm and polar angle division number of 6 well agrees with the reference result, which shows the validity of BACH. Using the pass line distance of 0.5 cm , the difference from the reference is only $0.05 \% \Delta k / k$, which can be neglected in design calculations.
The neutron spectrum calculated by BACH was compared with the reference result, and good agreement was obtained for individual pins. The within-assembly reaction rate distributions were also compared with the reference ones. Figure 4 compares the $\mathrm{Pu}-239$ fission and capture rate distributions. The GMVP results have relatively large standard deviations near the center of the assembly, and show a dip at the center. The present BACH does not produce such a problem, and leads to smooth results.



Fig 4 Normalized Reaction rate distributions

### 3.2 Color Set Calculation

Next we have applied BACH to a color set problem shown in Fig.5. The central assembly has a special pin at the center. The $k_{\text {eff }}$ calculated by GMVP and BACH are $1.36970 \pm 0.014 \%$ and 1.36953 respectively, and a good agreement is obtained.
The neutron spectra calculated are compared in Fig.6. It is noted that the GMVP could not calculate the low-energy spectrum at the center of the central assembly because of the absence of neutrons slowing down in the low energy range at the position.

BACH produced a reasonable result. This is a great advantage for the present method because one can accurately estimate reaction rates of small dosimeter foils.


Fig 5 Mini core for color set problem


Fig 6 Neutron spectra at the center of central assembly

## 4. Conclusion

The 3-D heterogeneous core calculation method has been introduced based on the characteristics method and the nodal transport method.
The heterogeneity of assemblies can be accurately taken into account by the characteristics method. Based on the method a computer code BACH has been made. The code was applied to the assembly calculations and the color set calculations. In both applications, the BACH results are in good agreement with the reference Monte-Carlo method. Furthermore, for the detailed information such as reaction rates of small samples, the BACH results were superior to the Monte-Carlo method.

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